The Case for Whole PI and Alternative Equations for Space, Mass, and the Periodic Table

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Abstract

A new series of equations for space, mass and the Periodic Table based on a common pattern is presented. Three equations for circular space represent the circumference (C), area (A), and volume (V) of a circle or sphere, which are mathematically equivalent to the conventional equations, specifically, $C=2\pi r$, $A=\pi r^2$, and $V=4/3\pi r^3$. The new equations incorporate a new understanding of pi, referred to as Whole PI to distinguish it from the classic understanding. A new symbol for Whole PI, ɸ, is presented and explained. Using Whole PI, the equations for the dimensions of space become $2ɸd^p/2p$ for the first dimension and $ɸd^p/2p$ for the others. It is further shown that the second mass, helium, stands in relation to the first mass, hydrogen, the same as the second dimension of space stands in relation to the first dimension of space, specifically, $H=2m^p/2p$ and $He=m^p/2p$, in which m equals the integer unit mass (m=1), the power signifies the atomic number (and therefore the number of electrons), and the denominator signifies the integer mass of the atom. Because of the similarity to the equations for dimensions of space, the elements may be referred to as dimensions of mass. Using the new equations, it is shown that the Periodic Table contains exactly ten dimensions of mass, and the other elements can be considered deconstructions of the ten dimensions of mass. Finally, an approximation is introduced for the accurate monoisotopic mass of hydrogen, specifically $(ɸ/3)^{1/6}$, which is 99.989% accurate to the observed monoisotopic mass of hydrogen.

Keywords: pi; tau; phi; Unit Simulacrum; Whole PI; Periodic Table

1. Introduction

This work arose from research into the use of liquid chromatography/mass spectrometry (LC-MS) to describe structural characteristics of triacylglycerols (TAGs) in fats and oils. To facilitate understanding of the ideas presented, it is helpful to retrace the evolution of the new concepts, reflected in the literature trail, from a very specific and targeted LC-MS application, through an update of those concepts, followed by generalization of the construct to be more widely applicable, then further generalization to development of a new tool and universal function, all the way through to application of the new tool to areas far outside the realm of LC-MS of TAGs.

In 2005, a construct called the Bottom Up Solution (BUS) to the Triacylglycerol Lipidome [1] was developed to derive structural information, related to the nutritional value of TAGs, from LC-MS data obtained on an instrument that employed atmospheric pressure chemical ionization mass spectrometry (APCI-MS). The BUS used ratios of the abundances of protonated molecule ions, $[MH]^+$, and diacylglycerol-like fragment ions, $[DAG]^+$, in APCI-MS mass spectra to determine structural characteristics of TAGs. Three Critical Ratios were identified that provided the desired structural information. The first Critical Ratio correlated the $[MH]^+/\Sigma[DAG]^+$ ratio to the degree of unsaturation (number of double bonds) in TAGs based on trends first reported by Byrdwell and Emken [2] in 1995. This relationship was modelled using a sigmoid function [3]. Polyunsaturated fats gave a high $[MH]^+/\Sigma[DAG]^+$ ratio, and saturated fats gave a $[MH]^+/\Sigma[DAG]^+$ ratio of essentially zero. The second Critical Ratio allows identification regioisomers of TAGs based on trends earlier reported by Mottram and Evershed [4], and Laakso and Voutilainen [5]. In other words, Critical Ratio 2 was used to identify the fatty acid (FA) in the middle, or *sn*-2 position (using stereospecific numbering, *sn*). Knowledge of regioisomers is important to human nutrition, since fat metabolism is regioselective, with preferential removal of FAs in the *sn*-1 and *sn*-3 positions during digestion. Regarding the third Critical Ratio, for two decades, no trends were reported for the $[1,2\text{-}DAG]^+$ versus $[2,3\text{-}DAG]^+$ fragments. However, the use of Critical Ratios allowed new insights that made it possible for trends to be identified for the first time [3].

The factors primarily responsible for the abundances of the $[1,2\text{-DAG}]^+$ and $[2,3\text{-DAG}]^+$ fragment ions were the degree of unsaturation and the grouping of unsaturated FAs either adjacent to each other or not. Thus, the three Critical Ratios provided new information necessary for structural characterization and quantification of TAGs by APCI-MS.

The great benefit of the Critical Ratios was that they also constituted a compact library of mass spectra. Not only did they provide the structural information desired at face value, but since it took fewer Critical Ratios to express the data than raw abundances in the mass spectra, they also represented a compressed data set. When the ratios were processed through the BUS, the original mass spectrum could be reproduced. Thus, the Critical Ratios also constituted a library of TAG mass spectra. The BUS from APCI-MS was later generalized and updated, by noticing the similarities in the Case classifications and simplifying them, to produce the Updated Bottom Up Solution (UBUS) [3]. Next, the construct was further generalized to apply equally well to atmospheric pressure photoionization (APPI) MS and electrospray ionization (ESI) MS of TAGs [6].

As the name states, and as described above, the Bottom Up Solution [1] and the Updated Bottom Up Solution [3] were developed from the bottom up, based on the foundation of Critical Ratios, and built up from those to allow the original mass spectra to be reproduced from the ratios. Once the BUS and UBUS were constructed in their entireties, the pattern behind these constructs could be seen and elucidated. It was found that for every ratio that was constructed, there existed the inverse ratio that could have been constructed, but was not. However, there are circumstances in which the inverse ratios might be more desirable than the ratio that was constructed. For instance, in ESI-MS (in contrast to APCI-MS), TAGs sometimes give only an ammonium or other adduct ions, with no $[\text{DAG}]^+$ fragments (unless some up-front collision induced dissociation (CID) energy is provided). In such cases, it would <u>not</u> be appropriate to use the $[\text{MH}]^+/\Sigma[\text{DAG}]^+$ ratio as Critical Ratio 1, since this could lead to division by zero, and an irrational value. In such cases, the $\Sigma[\text{DAG}]^+/[\text{MH}]^+$ would be the preferred Critical Ratio, so that the construct would remain bounded and rational, but still provide the desired structural information and still represent a compressed data set and compact library of mass spectra. These factors have been discussed in detail elsewhere [7], but the important point is that it was desirable to have a "top-down" solution that provided all options, whether selected or not selected, constructed or not constructed, real or unreal, rational or irrational, so that the best alternative for a particular MS application could be selected. Therefore, the BUS and UBUS were further generalized to produce the top-down solution that contained all possibilities of ratios and their inverses, known as the Simulacrum System (SS) for mass spectrometry of triacylglycerols [7].

A simulacrum is a construct that expresses the sum of two values (e.g. MS abundances) as a value and a ratio, and when one value is 1 (a requirement of MS), the solution simplifies to depend only on the ratio. When ratios are judiciously constructed (i.e. Critical Ratios) and processed in simulacrum solutions that are nested one, two or three levels deep, they provide structural information about TAGs and also produce compressed data sets like those described in the BUS [1] and UBUS [3, 6].

Note that in MS, ion abundances are usually expressed as percent relative abundances, and that percent means "per hundred", so 100% (100 per hundred) expressed as a pure ratio is 1. In mass spectra expressed as percent relative abundance, one peak, the base peak, is assigned a value of 100% (=1), and no ion can be greater than 100%. A simulacrum in which one value is 1 is called a Unit Simulacrum (US). Interestingly, every simulacrum solution contains a Unit Simulacrum inside the parentheses. Thus, a Unit Simulacrum is a fundamental component of all simulacra.

The process of identifying the "unit" in mass spectra over and over again, and the process of generalizing from the Bottom Up Solution to the top-down Simulacrum System, led to further generalization of the definition of any "unit". One characteristic of the Simulacrum System was that it is a function that applies not only to mass spectrometry of TAGs and other molecules, but also it is a universal function that applies to any number, letter, name, symbol, emoji, scribble, or any other designation that can be written in physical form. Therefore, for this report, we want to consider the nature of the "unit(s)" associated with the symbol for, and meaning of, a "unit pi", π, as well as a "unit circle", a "unit radius", and a “unit diameter”. Specifically, it is worthwhile to consider what is/are the “unit(s)” in the equations commonly used for circular dimensions of space.

A dictionary definition of pi is the ratio of the circumference of a circle to its diameter [8]. From this comes the mathematical equality for the circumference of a circle, $C = \pi \cdot d$. For a unit circle based on a unit diameter, the unit circle circumference is $C = \pi \cdot d_1$.

There is another definition for a circle, based on its radius, which is commonly used. The circumference of a circle based on radius is $C = 2\pi \cdot r$. For a unit circle based on a unit radius, the unit circle circumference is $C = 2\pi \cdot r_1$.

The two equations above show that there are two definitions for a unit circle: one based on diameter and one based on radius. Thus, for a unit circle, $C = \pi \cdot d_1 = 2\pi \cdot r_1$, or $C = \pi \cdot 1_d = 2\pi \cdot 1_r$. Of course, whatever the unit diameter is, a unit radius is ½ of a unit diameter based on the same units, since d=2r and r=d/2. In the Unit Simulacrum solutions for mass spectrometry, the goal was always to classify the Critical Ratio to determine which value was “carrying the unit”, so the “1” on the outside of the parentheses could be cancelled out and ignored, leaving the simplified simulacrum solution. In the above unit circle equations, the “1” can never be cancelled out and ignored, otherwise we arrive at the contradiction, or mathematical paradox, that $\pi = 2\pi$, or 3.14159… = 6.283185…. The unit circle has two different definitions, one based on “r” and one based on “d”. The name of the defining unit must accompany π, otherwise one cannot determine the unit that the circle is based on. See Supplemental Figure 1 for an explanatory analogy.

Another way to look at it is to consider the value of circumference = 10π (or any other specific multiple of π). From the designation 10π alone, it is impossible to distinguish whether this is a circle based on radii (radius of 5) or a circle based on a diameter (diameter of 10). Both are equal and true. There are two possible definitions for the same symbolic value, 10π.

We can look at this another way. The above discussion was about a unit circle, not a unit π. For a circle based on radius, the circumference is 2π, so a single unit π is half of a circle. In contrast, for a circle based on diameter, the circumference is π, so a single unit π is a whole circle. Thus, a single unit π

represents either a half a circle or a whole circle, depending on whether the defining unit is a radius or a diameter. In other words, the symbol π has two different unit definitions. The fact that one single symbol, π, represents two different entities, either half of a circle or a whole circle, and that without the defining unit always accompanying π we can arrive at the mathematical paradox $2\pi = \pi$, can be called the "pi symbol paradox".

Of course, this paradox also plays out in calculations for the second dimension ($r \cdot r$ or $d \cdot d$) and third dimension ($r \cdot r \cdot r$ or $d \cdot d \cdot d$). The equations based on radius are the well-known $\pi \cdot r^2$ and $4/3\pi \cdot r^3$. The equations based on diameter are the less well-known $\pi \cdot d^2/4$ and $\pi \cdot d^3/6$. If one were to see only the value for area of $A = \pi$, one could not determine from this information alone whether it was the area of a circle based on radius, with a radius of 1, or the area of a circle based on diameter, with a diameter of 2. Both are mathematically equal, but represent different circles based on different units. Nevertheless, because of the pi symbol paradox, these two cannot be differentiated without being explicitly told what the defining unit is. Similarly, if one were to see only the value for volume, $V = 36\pi$, one could not determine from this information alone whether it was the volume of a sphere based on radius, with a radius of 3, or the volume of a sphere based on diameter, with a diameter of 6. Both are mathematically equal, and give a volume of 36π, but represent different spheres based on different units. Because of the pi symbol paradox, two different spheres based on different units give the same symbolic value for a volume. This report discusses this issue in greater detail below, and presents an unambiguous solution to the pi symbol paradox.

Before proceeding, it is important to mention that for more than a decade there has been a movement underway to implement a new definition, called τ, which represents a complete circle in terms of radians, or $\tau = 2\pi = 6.283185\ldots$, which is the ratio of a circle's circumference to its radius. As discussed in Scientific American [9]: "The crux of the argument is that pi is a ratio comparing a circle's circumference with its diameter, which is not a quantity mathematicians generally care about. In fact, almost every mathematical equation about circles is written in terms of r for radius. Tau is precisely the number that connects a circumference to that quantity… At its heart, pi refers to a semicircle, whereas tau refers to the circle in its entirety." The fact that $2\pi\cdot r$ is really two times a semicircle was summarized in a 2013 feature on the PBS Newshour in a quote by Mike Keith [10]: "It's like reaching your destination and saying you are twice halfway there".

While I was not aware of the proposal of τ and its base of support (self-proclaimed "tauists") before using the Unit Simulacrum to reconsider the nature of π and the "unit" associated with it, I separately came to a similar conclusion about the value of considering the whole circle, instead of only half. While I have not adopted the opinion expressed elsewhere that "pi is wrong" [11], I did arrive at a new formulation that may prove beneficial for modelling the dimensions of space, mass, and the Periodic Table, to complement, not replace, the classic understanding of π.

2. Results

The Unit Simulacrum for π, SimSum(1,π), which equals $\Sigma(1+\pi)$, where the mathematical sum is expressed as a value and a ratio, is shown in Figure 1. In this example, the "unit" is a unit radius, $r = 1$, or r_1. As was done in the case of MS, we choose the solutions that have the 1 multiplying outside the parentheses, so that it cancels out, leaving the simplified simulacrum solutions: $1+(\pi/1)$ or $1+1/(1/\pi)$. The Unit Simulacrum for π can be called the Pi Unit Simulacrum (PiUS). Since π has a well-known value of 3.14159…, which is ≥ 1, the PiUS is always Case 2 (note that it was previously demonstrated [7] that the Simulacrum provides all solutions, whether selected or unselected, observed or unobserved, real or unreal, rational or irrational). As with the BUS and UBUS for MS of TAGs, the simplified Unit Simulacrum solutions always end up being 1+ratio or 1+1/ratio.

Let us further examine the ratios $(\pi/1)$ and $(1/\pi)$ that appear in the PiUS solutions. As stated above, the value that equals 1, the unit, is r_1. When we draw the unit and π, Figure 2A, we see that it has r_1 and half of a circle. Thus, while Fig. 2A does show a unit radius, and a unit (one) π, it does <u>not</u> show a unit (one) circle; it shows a ½ circle. Since the BUS and UBUS for MS of TAGs were based on judiciously

Simulacrum Sum (1, π) = SimSum(1, π) = $\sum_{2\text{-}0\text{-}1\text{-}\pi}(1+\pi) =$

Possibilities to Observe:

$$\begin{array}{cc} 1 & \pi \\ \left(\frac{1}{\pi}\right) & \left(\frac{\pi}{1}\right) \end{array}$$

$$\left\{\begin{array}{lcl}
\text{Case 1: } \pi \le 1, 1 \ge \pi, \left(\frac{\pi}{1}\right) \le 1, \left(\frac{1}{\pi}\right) \ge 1; & & \\
\pi\left(1+\frac{1}{\left(\frac{\pi}{1}\right)}\right) & \text{or} & 1\left(1+\left(\frac{\pi}{1}\right)\right) \\
\pi\left(1+\left(\frac{1}{\pi}\right)\right) & \text{or} & 1\left(1+\frac{1}{\left(\frac{1}{\pi}\right)}\right) \\
\text{Case 2: } \pi \ge 1, 1 \le \pi, \left(\frac{\pi}{1}\right) \ge 1, \left(\frac{1}{\pi}\right) \le 1; & & \\
\pi\left(1+\frac{1}{\left(\frac{\pi}{1}\right)}\right) & \text{or} & 1\left(1+\left(\frac{\pi}{1}\right)\right) \\
\pi\left(1+\left(\frac{1}{\pi}\right)\right) & \text{or} & 1\left(1+\frac{1}{\left(\frac{1}{\pi}\right)}\right)
\end{array}\right\}$$

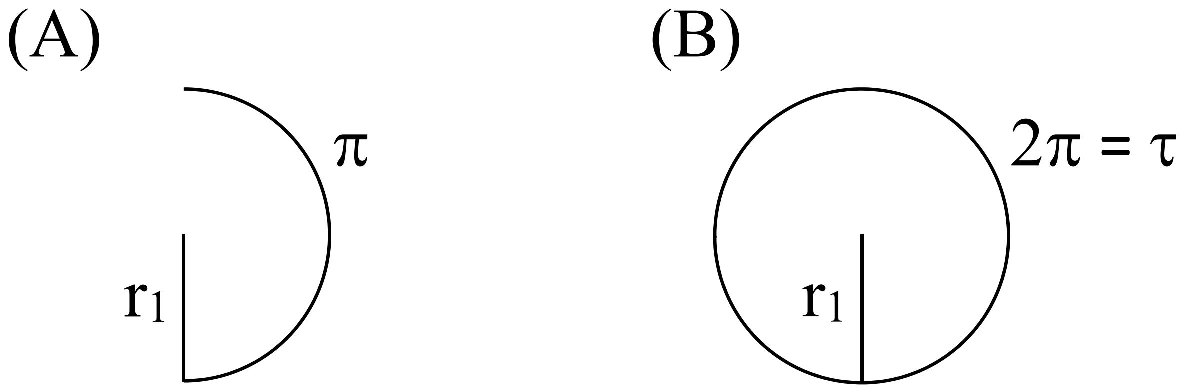
Figure 1. The π Unit Simulacrum (PiUS), where one value is specified to be 1 and the other value is specified to be π. This is Case 2 by default, since π ≥ 1. A simulacrum is composed of the Simulacrum Sum, four Possibilities to Observe, two Cases, and eight solutions.

selecting (constructing) Critical Ratios, we can ask the question of π: "Is there another ratio we could use?"

Obviously, from the discussion of τ above, the answer is yes. The proponents of τ advocate constructing the ratio of the unit radius, r_1, to the unit circle, 2π. If 2π is substituted into Fig.1 for π, and the solutions that contain the unit radius, r_1, outside the parentheses are chosen as before, to allow the 1 to cancel out and simplify the solutions, then the simulacrum solutions for $\mathrm{SimSum}(1,2\pi) = \Sigma(1+2\pi)$ become $1+(2\pi/1)$ or $1+1/(1/2\pi)$, taken from Case 2, since $2\pi \geq 1$. Again, we can graphically visualize the two ratios $(2\pi/1)$ and $(1/2\pi)$ as shown in Fig. 2B. The figure based on the constructed ratio 2π has the advantage that it now shows a unit radius and a unit circle, but it does <u>not</u> show unit π. Instead, it shows 2π, which makes the unit circle. In this case, the circumference of the unit circle is $2\pi\cdot r_1$, or just 2π, which equals 6.283185…

If instead of using 2π, we substitute τ into the Unit Simulacrum for π in Fig. 1, and again allow the unit radius outside the parentheses to cancel out, we obtain the two Case 2 (since $\tau \geq 1$) solutions based on the two ratios that were possible to observe. The two simulacrum solutions to the $\mathrm{SimSum}(1,\tau) = \Sigma(1+\tau)$ are $1+(\tau/1)$ and $1+1/(1/\tau)$. These ratios are also visualized in Fig. 2B. The figure based on constructing the ratio of τ to r_1 now has the benefits of showing a unit radius, a unit (one) τ, and a unit circle. In this case, the circumference of the unit circle is $\tau\cdot r_1$, or simply τ, which equals 6.283185…

In Figure 3 we also have a unit circle, but now a unit diameter, d_1, instead of a unit radius, r_1. The circumference of this unit circle is $\pi\cdot d_1$, or simply π. Comparing Fig. 2 to Fig. 3 inescapably brings us to the "pi symbol paradox", which is that the symbol for pi has two different unit definitions based on two different unit circles. Based on a unit radius, the unit circle is $2\pi\cdot r_1$. Based on a unit diameter, the unit circle is $\pi\cdot d_1$. Thus, the same one whole unit circle is both $2\pi\cdot r_1$ and $\pi\cdot d_1$. Therefore, the units, r_1 or d_1, can never be cancelled out or ignored, otherwise we obtain $2\pi = \pi$, or $6.283185\ldots = 3.14159265\ldots$



$$\left(\frac{\pi}{r_1}\right)=\left(\frac{\pi}{1}\right)$$

or

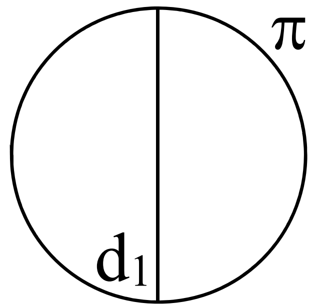
$$\left(\frac{r_1}{\pi}\right)=\left(\frac{1}{\pi}\right)$$

$$\left(\frac{2\pi}{r_1}\right)=\left(\frac{2\pi}{1}\right)=\left(\frac{\tau}{r_1}\right)=\left(\frac{\tau}{1}\right)$$

or

$$\left(\frac{r_1}{2\pi}\right)=\left(\frac{1}{2\pi}\right)=\left(\frac{r_1}{\tau}\right)=\left(\frac{1}{\tau}\right)$$

Figure 2. Unit radius definitions. **(A)** Graphical representation of the relationship between a unit radius ($r = 1$), r_1, and π, showing the ratios of π to r_1 and r_1 to π. **(B)** Graphical representation of the relationship between a whole circle based on radii and a unit radius, r_1, showing the ratios of 2π to r_1 and r_1 to 2π, as well as τ to r_1 and r_1 to τ. The whole circle in (B) has a circumference of 2π, or 6.283185…, or τ.



$$\left(\frac{\pi}{d_1}\right)=\left(\frac{\pi}{1}\right)$$

or

$$\left(\frac{d_1}{\pi}\right)=\left(\frac{1}{\pi}\right)$$

Figure 3. Definition of π based on diameter. Graphical representation of the relationship between a unit diameter ($d = 1$), d_1, and π, showing the ratios of π to d_1 and d_1 to π. The whole circle has a circumference of π, or 3.14159265…

Thus, unless the "r" or the "d" always accompanies π, it cannot be known with certainty which whole circle π refers to.

To solve the pi symbol paradox, a new symbol for pi was developed, based on a whole unit diameter and a whole unit circle, which is called Whole PI. Figure 4 shows the components of the new symbol for Whole PI. Since the symbol is intended to represent a whole unit circle based on a diameter, the symbol contains a whole circle, ○, and a diameter, | . However, to emphasize that the symbol represents all three dimensions of space, 3-D, the symbol includes three diameters representing the three dimensions, |||, Fig. 4A. Furthermore, to make the symbol two-dimensional so it can easily be written, the diameters are arranged in a ratio of 1:2, which also reflects the relationship between radii and diameters, which is 1:2, Fig. 4B. Thus, the symbol for Whole PI is ɸ, Fig. 4C, and this clearly and unambiguously differentiates it from the original, or classic pi, which can now be used exclusively for representing the whole unit circle, 2π, defined based on a unit radius. Having two symbols for the two different circles based on two different unit definitions eliminates all uncertainty and solves the pi symbol paradox, because the symbol itself carries an indication of the defining unit.

If we construct the Unit Simulacrum for ɸ, we can select the two simplified simulacrum solutions, 1+(ɸ/1) and 1+1/(1/ɸ). The ratios (ɸ/1) and (1/ɸ) used in these solutions are represented in Figure 5. From this figure, it is now perfectly clear that the unit ɸ, based on the unit d_1, cannot be confused with π from Fig. 2 or Fig. 3.

2.1. Dimensions of Space

Using Whole PI for the equations given in the introduction for the three dimensions based on diameter, we have ɸd, ɸd^2/4, and ɸd^3/6. It is easy to see from the second and third dimensions that they obey a common pattern, which is ɸd^p/2p, shown in Figure 6B and Fig. 6C. In this pattern we can see that the power is the dimension number, which is the same as the number of diameters in that dimension. The denominator, 2p, reflects the numbers of radii in the area and volume, which are 4 and 6, respectively.

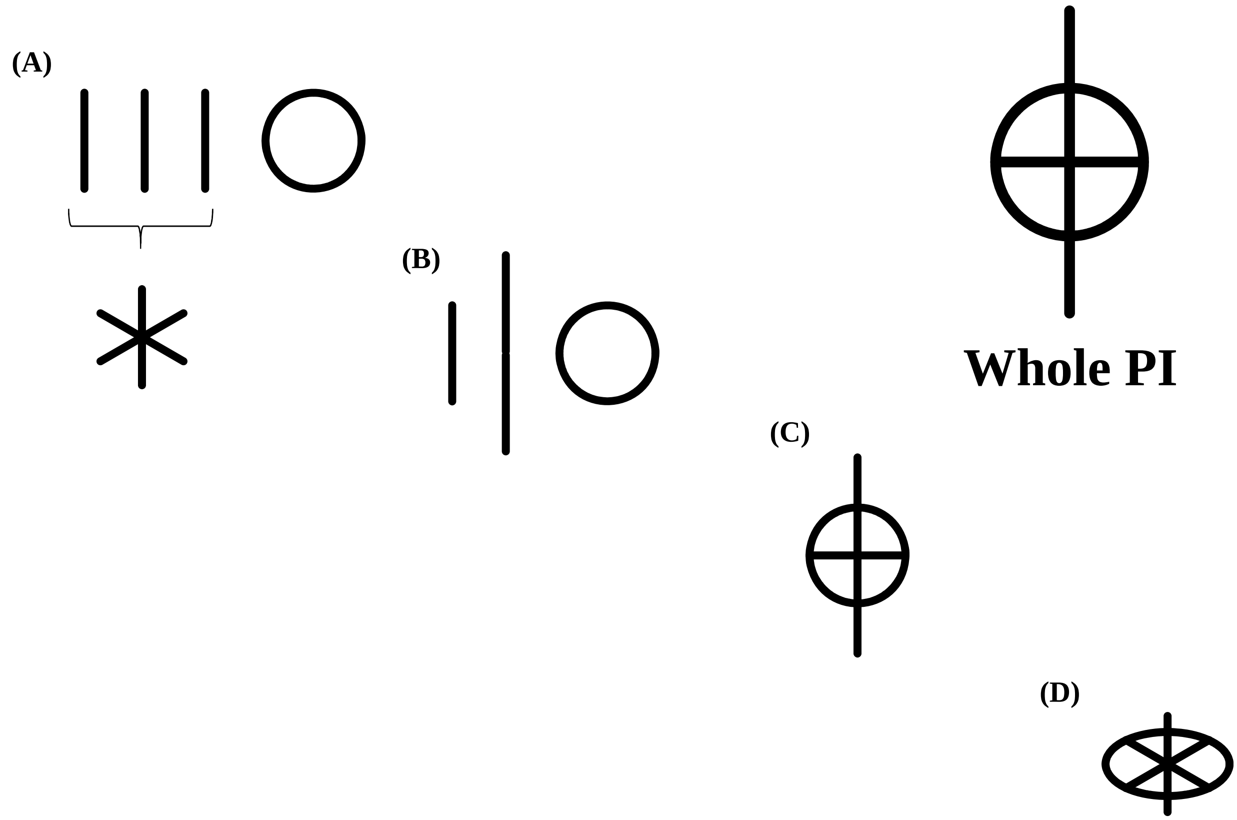
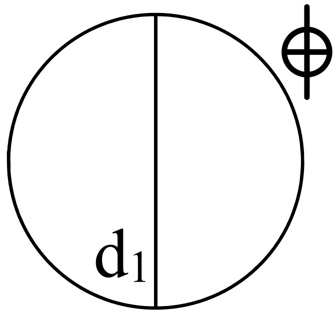


Figure 4. Construction of the new symbol for Whole PI. **(A)** It contains three diameters, |||, representing the three dimensions of space, and contains a whole unit diameter circle, ○. **(B)** The ratio of the first diameter to the two others, 1/2, is the same as the ratio of a radius to a diameter, providing self-similarity. **(C)** The symbol for Whole PI is two-dimensional, ⌽, **(D)** but the third dimension can be envisioned from (A), ⊗.



$$\left(\frac{\phi}{d_1}\right) = \left(\frac{\phi}{1}\right)$$

or

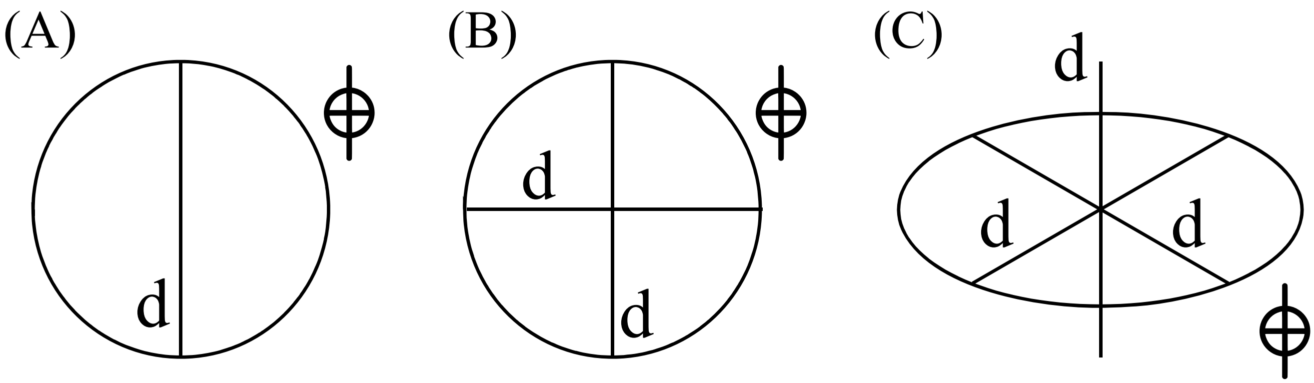
$$\left(\frac{d_1}{\phi}\right) = \left(\frac{1}{\phi}\right)$$

Figure 5. Unit diameter Whole PI circle. Graphical representation of the relationship between a unit diameter ($d = 1$), d_1, and ϕ, showing the ratios of ϕ to d_1 and d_1 to ϕ. The whole circle has a circumference of ϕ, or 3.14159265…

The first dimension is unique. If we construct the equation in the same form as the second and third dimensions, to raise "d" to the power of "p" and then show 2p in the denominator to reflect the number of radii in the first dimension, we find that the equation for the first dimension has an additional "2" in the numerator, to give $2ɸd^p/2p$. We can think of the first of anything as being unique because it serves two purposes: it defines a new category, and also represents the first unit in that category. Every other occurrence of the defined thing is another unit in that category. Pragmatically, we know that the first of anything is different and special, such as a first literature report, first car, first job, first house, etc. Thus, it is not entirely unprecedented or unexpected that the First Dimension has a "2" in the numerator that makes it unique, while the rest of the equation follows the same pattern as the other dimensions. Together, we have the equations for the three dimensions as $2ɸd^p/2p$, $ɸd^p/2p$, and $ɸd^p/2p$ for the first, second, and third dimensions of space. These simplify to being one equation for the first dimension, $2ɸd^p/2p$, and a slightly different single equation for the other dimensions, $ɸd^p/2p$.

As an example, we can consider a circle with a diameter of 12 units (so a radius of 6 units): the circumference is $C = 2ɸd^p/2p = 2ɸ(12)^1/2*1 = 12ɸ$. Or using the classic approach: $C = 2\pi r = 2(6)\pi = 12\pi$. These two circumferences, $12ɸ$ and 12π are mathematically equal, but can now be clearly differentiated, and there is no ambiguity that $12ɸ$ is the circumference of the circle based on diameter, with $d = 12$; and 12π is the circumference of the circle based on radius, with $r = 6$. The uncertainty discussed in the Introduction is eliminated, and the pi symbol paradox is unambiguously resolved. Whereas having two unit definitions for classic pi led to the paradox $2\pi = \pi$, having two different symbols for the two different defined circles leads to $2\pi = ɸ$, in which the two symbols carry the meaning that they are based on two different "units", but they both represent a whole unit circle.

Similarly, the area is given by $A = ɸd^p/2p = ɸ(12)^2/2*2 = 36ɸ$ using the new equations. This is mathematically equal to, but clearly differentiated from the classic approach, which equals $A = \pi r^2 = \pi(6)^2 = 36\pi$. Thus, the two approaches give equivalent numerical solutions that are clearly distinguishable.



$$\left(\frac{2\phi d^{p_1}}{2p}\right) \quad \text{or} \quad \frac{2\phi d^1}{2} = \phi d$$

$$\left(\frac{\phi d^{p_2}}{2p}\right) \quad \text{or} \quad \frac{\phi d^2}{4}$$

$$\left(\frac{\phi d^{p_3}}{2p}\right) \quad \text{or} \quad \frac{\phi d^3}{6}$$

Figure 6. Whole PI equations for dimensions of space. **(A)** Graphical representation of, and equation for, the relationship between any diameter, d, and Whole PI, ϕ, for the First Dimension of space. **(B)** Graphical representation of, and equation for, the relationship between any diameter, d, and Whole PI, ϕ, for the second dimension of space. **(C)** Graphical representation of, and equation for, the relationship between any diameter, d, and Whole PI, ϕ, for the third dimension of space. Notice that the number in the denominator (2p) represents the number of radii in each dimension. The First Dimension is unique, and has a two in the numerator, $2\phi d^p/2p$. The equations for the other dimensions are identical, $\phi d^p/2p$, differentiated only by the value of p, representing the dimension number, which also equals the number of diameters.

Likewise, the volume for the same circle is also given by $V = \phi d^p/2p = \phi\ (12)^3/2*3 = 288\phi$ using the new equations. This is mathematically equal, but clearly differentiated from the classic approach, which equals $V = 4/3\pi r^3 = 4/3\pi(6)^3 = 288\pi$. Thus, the two approaches give equivalent numerical solutions that are clearly distinguishable as one being based on diameter, the other being based on radius.

The alternative equations for dimensions of space based on Whole PI have several advantages and convey more information at face value. First, it is advantageous that it is very easy to see the pattern behind the equations $2\phi d^p/2p$, $\phi d^p/2p$, and $\phi d^p/2p$, especially since the equations for the second and third dimensions are identical. Second, the first dimension is designated as unique compared to all other dimensions, by having a factor of 2 to distinguish it from the other dimensions. Third, all equations for dimensions now reflect the duality that every diameter contains two radii, and the number of radii in each dimension is explicitly given in the denominator of every equation, in the factor "2p".

2.2. Dimensions of Mass

It soon became apparent that the second dimension of space stands in relation to the first dimension of space in the same way that the second mass in the Periodic Table, helium, stands in relation to the first mass, hydrogen. That is to say that an equation for hydrogen can be put in the same form as the equation for the first dimension of space, by replacing "d" with "m", shown in Figure 7, while an equation for helium can be put in the same form as the second dimension of space by replacing "d" with "m". Integer masses are considered first, which do not require the use of Whole PI, while an approximation for monoisotopic masses that incorporates ϕ and/or π is discussed further below. Using the same pattern as the dimensions for space, the first mass is $2m_1{}^p/2p$, while the second mass is $m^p/2p$, where "m_1" is a unit integer mass, m_1, representing a single proton or neutron, the "p" represents the atomic number, and the denominator represents the total integer mass. Or more precisely, the total integer mass is the inverse of the equations given below. Thus, $2m^1/2$ (= $m^1/1$) and $m^2/4$ are similar in form to $2\phi d^1/2$ (= $\phi d^1/1$) and $\phi d^2/4$.

(A) $$\left(\frac{2m^{p_1}}{2p}\right)$$

or

$$\frac{2m^1}{2} = \frac{m^1}{1}$$

(B) $$\left(\frac{m^{p_2}}{2p}\right)$$

or

$$\frac{m^2}{4}$$

(C) $$\left(\frac{m^{p_3}}{2p+1}\right)$$

$$\frac{m^3}{7}$$

(D) $$\left(\frac{m^{p_4}}{2p+1}\right)$$

$$\frac{m^4}{9}$$

(E) $$\left(\frac{m^{p_5}}{2p+1}\right)$$

$$\frac{m^5}{11}$$

(F) $$\left(\frac{m^{p_6}}{2p}\right)$$

$$\frac{m^6}{12}$$

(G) $$\left(\frac{m^{p_7}}{2p}\right)$$

$$\frac{m^7}{14}$$

(H) $$\left(\frac{m^{p_8}}{2p}\right)$$

$$\frac{m^8}{16}$$

Figure 7. Dimension model of mass. **(A)** Equation for the first unit mass, m, which contains the atomic number as the power, p, and the integer mass as the denominator. **(B)** Equation for the second mass, m, which contains the atomic number as the power and the integer mass as the denominator. **(C)** Equation for the third mass, m, which contains the atomic number as the power and the integer mass as the denominator. **(D** and **E)** Equations for the fourth and fifth masses, which are deconstructions. **(F** to **H)** Equations for sixth, seventh, and eighth elements, which are the third, fourth, and fifth dimensions of mass.

Based on the same form of equation for dimensions of space and mass, we could hypothesize that masses that are in this form of equation may be considered “dimensions of mass”. In the same way that the dimensions of space contain the number of subcomponents (radii) in the denominator of all equations, the dimensions of mass contain the number of subcomponents (protons + neutrons) in the denominator of all equations. Using the dimension form of equations, there are exactly ten dimensions of mass in the Periodic Table, enumerated below.

The third mass, lithium, incorporates another principle discussed in the report of the Simulacrum System for mass spectrometry. A Unit Simulacrum, based on 1+ratio, is the same as a ratio+1. Adding 1 to something can said to be incrementing that something. Conversely, adding 1 to the denominator of a ratio, such as (1/1), can be said to be decrementing that something, as previously discussed [7]. Thus, the first increment of 1 is 1+1=2, and the first decrement of 1/1 is 1/(1+1) = ½. The first decrement is also referred to as the first deconstruction. From the sections above, we can see that a radius, r, is the first deconstruction of a diameter, d, or r = d/(1+1), r = d/2.

Since $m_1^p/2p$ is analogous to a dimension of mass, just as $ɸd^p/2p$ was a dimension of space, then the first increment in the denominator is a deconstruction of that dimension of mass (again, considering integer masses, with a factor incorporating ɸ discussed further below). Lithium follows the form of equation $m_1^p/(2p+1)$, indicating that it is a deconstruction of the previous dimension of mass. In fact, lithium, beryllium, and boron all follow the form $m_1^p/(2p+1)$. The principles in Fig. 7 are followed through the penultimate dimension of mass, sulfur, which is $m_1^p/2p$, where p=16, as follows:

H_1: $2m_1^1/2p$

He_2: $m_1^2/2p$

$Li_{2\text{-}1}$: $m_1^3/(2p+1)$, $Be_{2\text{-}2}$: $m_1^4/(2p+1)$, $B_{2\text{-}3}$: $m_1^5/(2p+1)$

$C_{3\text{-}3}$: $m_1^6/2p$

$N_{4\text{-}3}$: $m_1^7/2p$

$O_{5\text{-}3}$: $m_1^8/2p$

$F_{5\text{-}4}$: $m_1^9/(2p+1)$

$Ne_{6\text{-}4}$: $m_1^{10}/2p$

$Na_{6\text{-}5}$: $m_1^{11}/(2p+1)$

$Mg_{7\text{-}5}$: $m_1^{12}/2p$

$Al_{7\text{-}6}$: $m_1^{13}/(2p+1)$

$Si_{8\text{-}6}$: $m_1^{14}/2p$

$P_{8\text{-}7}$: $m_1^{15}/(2p+1)$

$S_{9\text{-}7}$: $m_1^{16}/2p$

$Cl_{9\text{-}8}$: $m_1^{17}/(2p+1,3)$, $Ar_{9\text{-}9}$: $m_1^{18}/(2p+4)$

$K_{9\text{-}10}$: $m_1^{19}/(2p+1)$

$Ca_{10\text{-}10}$: $m_1^{20}/2p$

Since the power (exponent) is the atomic number, it includes both dimensions and deconstructions. To differentiate the dimension number and number of deconstructions, a nomenclature can be adopted that distinguishes the dimensions from the deconstructions, specifically 2 dash 1, 2-1, for lithium, which specifies the dimension number followed by the deconstruction number. This nomenclature is shown as the subscript for each element symbol in the list above. For example, using this nomenclature sulfur is the 9th dimension of mass and contains 7 deconstructions, 9-7, while calcium, element 20, is the 10th (and final) dimension of mass and contains ten deconstructions, 10-10. The element number, the single number for the power of the unit mass, can be replaced with the dimension-deconstruction notation, and the atomic number is the sum of dimensions and deconstructions. But for this first report, the atomic number is listed as a single value in the powers shown.

Chlorine$_{9\text{-}8}$ and argon$_{9\text{-}9}$ exhibit different patterns, while potassium$_{9\text{-}10}$ is again a simple deconstruction, $m_1^{19}/(2p+1)$. Chlorine is unique, since it has a molar mass of ~35½, being the isotope-weighted average of $m_1^{17}/(2p+1)$ and $m_1^{17}/(2p+3)$, and is the first element to exhibit such a large second isotope that differs by *two* masses from the monoisotopic mass (~1/3 of primary isotope). Argon is unique, since it is the first anomaly in the periodic table, being the first element to have a mass larger than the following element. Before addressing these elements, it is worthwhile to mention the shortcomings of the simple model presented here.

While the simple model for dimensions of mass and deconstructions provides information about the dimension number, deconstruction number, atomic numbers, and masses of elements, it does not incorporate any factors related to several other physically observed phenomena, specifically: 1) isotope distribution, 2) ferromagnetism, 3) radioactivity and others (metals, etc.). Because of these factors, the simple model presented here is only a starting point for thinking of a fuller theoretical description of atomic elements, not a final all-encompassing model. Of course, there is no other extant model that presents masses in the same form as dimensions of space, and certainly not one that incorporates the above factors, so the dimension model of mass (DMM) does have benefits, despite its shortcomings.

Chlorine is not a simple (2p+1) deconstruction, since it has two abundant isotopes, $m_1^{17}/(2p+1)$ and $m_1^{17}/(2p+3)$ in a ratio of ~3:1. The isotope (2p+3) could be expressed as $(2p+2f(x_1)+1)$, where $f(x_1)$ is a different function (with first value equal to 1), not the atomic number, but related to some other phenomenon, such as those mentioned above (e.g. isotope distribution). Chlorine has a molar mass of very close to 35 ½, represented by $m^{17}/(2p+1+(1/(1+1)))$. Thus, because chlorine is the first element to exhibit such a substantial amount of a higher order of deconstruction than simply 1/(2p+1), it is not included with the other simple 1/(2p+1) deconstructions.

Argon similarly exhibits a higher level of deconstruction, but now as its primary isotope, having an equation of $m_1^{18}/(2p+4)$. The deconstruction 1/(2p+4) can be expressed either as: 1) $1/(2p+2f(x_2))$, where $f(x_2)$ is the same function as mentioned for chlorine, but with a value of 2, or 2) $1/(2p+1+(2f(x_1)+1))$, where $2f(x_1)+1$ is the same function of deconstruction as for chlorine, or 3) $1/(2p+1+(2f(y_1)+1))$, where $2f(y_1)+1$ is a different function of deconstruction related to some other characteristic or phenomenon (ferromagnetism, radioactivity, etc.), with its first value equal to 1. For now, it is indeterminate which of these possibilities (or another not listed) gives rise to the 1/(2p+4), so argon is listed in Figure 8 simply as $m_1^{18}/(2p+4)$.

H $\frac{2m^{1}}{2p}$																	He $\frac{m^{2}}{2p}$
Li $\frac{m^{3}}{2p+1}$	B $\frac{m^{4}}{2p+1}$											B $\frac{m^{5}}{2p+1}$	C $\frac{m^{6}}{2p}$	N $\frac{m^{7}}{2p}$	O $\frac{m^{8}}{2p}$	F $\frac{m^{9}}{2p+1}$	Ne $\frac{m^{10}}{2p}$
Na $\frac{m^{11}}{2p+1}$	Mg $\frac{m^{12}}{2p}$											Al $\frac{m^{13}}{2p+1}$	Si $\frac{m^{14}}{2p}$	P $\frac{m^{15}}{2p+1}$	S $\frac{m^{16}}{2p}$	Cl $\frac{m^{17}}{2p+1,3}$	Ar* $\frac{m^{18}}{2p+4}$
K $\frac{m^{19}}{2p+1}$	Ca $\frac{m^{20}}{2p}$	Sc $\frac{m^{21}}{2p+3}$	Ti $\frac{m^{22}}{2p+4}$	V $\frac{m^{23}}{2p+5}$	Cr $\frac{m^{24}}{2p+4}$	Mn $\frac{m^{25}}{2p+5}$	Fe $\frac{m^{26}}{2p+4}$	Co $\frac{m^{27}}{2p+5}$	Ni* $\frac{m^{28}}{2p+2,4}$	Cu $\frac{m^{29}}{2p+5,7}$	Zn $\frac{m^{30}}{2p+4,6}$	Ga $\frac{m^{31}}{2p+7,9}$	Ge $\frac{m^{32}}{2p+10,8}$	As $\frac{m^{33}}{2p+9}$	Se* $\frac{m^{34}}{2p+12,10}$	Br $\frac{m^{35}}{2p+9,11}$	Kr $\frac{m^{36}}{2p+12,14}$
Rb $\frac{m^{37}}{2p+11,13}$	Sr $\frac{m^{38}}{2p+12}$	Y $\frac{m^{39}}{2p+11}$	Zr $\frac{m^{40}}{2p+10,14}$	Nb $\frac{m^{41}}{2p+11}$	Mo $\frac{m^{42}}{2p+14,12}$	Tc $\frac{m^{43}}{2p+13}$	Ru $\frac{m^{44}}{2p+14,16}$	Rh $\frac{m^{45}}{2p+13}$	Pd $\frac{m^{46}}{2p+14,16}$	Ag $\frac{m^{47}}{2p+13,15}$	Cd $\frac{m^{48}}{2p+18,16}$	In $\frac{m^{49}}{2p+17}$	Sn $\frac{m^{50}}{2p+20,18}$	Sb $\frac{m^{51}}{2p+19,21}$	Te* $\frac{m^{52}}{2p+26,24}$	I $\frac{m^{53}}{2p+21}$	Xe $\frac{m^{54}}{2p+24,21}$

Figure 8. The Periodic Table of the Elements described using the dimensions of mass model, with the ten dimensions of mass in bold boxes. The power to which the integer unit mass, m_1, is raised represents the atomic number and the denominator represents the integer mass. When multiple isotopes are abundant (>~20%), the major isotope is listed first, with the second most abundant in smaller text. Anomalous masses are marked with asterisks. In the same way that r = d/(1+1) shows that the first increment in the denominator is a deconstruction, the values in the denominator indicate the level of deconstruction.

After argon, potassium is the last of the 1/(2p+1) deconstructions, being $m_1^{19}/(2p+1)$. Based on the template equation for dimensions, $m_1^p/2p$, calcium is the tenth and final dimension of mass, with $m_1^{20}/2p$. All other elements after calcium are deconstructions of the tenth dimension of mass, which are shown in Figure 8. The deconstructions become increasingly large, indicating increasing index values for f(x) and the likely presence of other functions, such as f(y), both related to characteristics or phenomena that are not incorporated into the simple initial model based only on dimensions of mass and first deconstructions, 1/(2p+1). The table in Fig. 8 was not extended past xenon since additional understanding of the phenomena that lead to larger values of deconstruction (e.g., $f(x_1)$, $f(y_1)$,…) is needed. Nevertheless, the dimension model of mass does provide new insight into a previously unreported pattern behind the Periodic Table that reflects a similarity between mass and dimensions of space. Even if used only for integer masses, the DMM provides new insights into the anomalies in the Periodic Table, as well as other trends, discussed below.

It is possible to use the same pattern given for space and mass above, incorporating $2x^p/2p$, $x^p/2p$, and $x^p/(2p+1)$, to further extend the model to provide approximations for observed monoisotopic masses.

2.3. The 99.989% Solution

While the equations for mass and the periodic table above did follow the form of dimensions and deconstructions and did provide information about the atomic number and integer mass (or the integer + ½ in the case of chlorine), they did not approximate the observed accurate monoisotopic masses. They also did not include ɸ or π; which is to say, they did not reflect circularity. The simplest whole number ratio to ɸ, raised to a simple power that produces the closest value to the accurate monoisotopic mass of hydrogen is $(\phi/3)^{1/6}$ (based on spreadsheet calculations). This constant has a value of 1.00771588137…, which is 99.989% accurate to the monoisotopic mass of hydrogen, which is 1.00782503223(9) [12]. This represents a difference of -108.3 ppm. However, simply multiplying each mass ($2p_m$, $2p_m+1$, etc., where p_m is the dimension+deconstruction atomic number) times $(\phi/3)^{1/6}$ does not provide a good enough model for all other masses. A better model, given below, is found by nesting the exponent, 1/6, into the same form of equations described for the first dimension, other dimensions, and deconstructions of mass, described in Fig. 7.

The better model is found by nesting the exponent (1/6) into the general form of dimension and deconstruction equations: $2x^p/2p$, $x^p/2p$, and $x^p/(2p+1)$, where x = (1/6). The (1/6) may represent the three dimensions of space in which mass operates, $d_1{}^p/2p_3 = 1^3/6$, or other factor including both space and mass, discussed further in the Electronic Supplementary Material. All possibilities proposed for the numerators below give a factor that reflects both mass and the three dimensions of space in which mass operates, resulting in a space-mass function. Then, the space-mass function is divided by the equation for each integer mass equation, $2m^p/2p$, $m^p/2p$, and $m^p/(2p+1)$, as given in Fig. 7 and Fig. 8, to yield the model monoisotopic mass for each element. For instance, for the first mass, hydrogen = H_1, the power, 1/6, is incorporated into the first dimension of mass equation as follows:

$$\mathrm{H}_1 = \frac{\left(\frac{\phi}{3}\right)^{\frac{2\left(\frac{1}{6}\right)^{p_1}}{2\boldsymbol{p_m}}}}{\frac{2\boldsymbol{m}_1^{\boldsymbol{p}}}{2\boldsymbol{p_m}}} = \frac{\left(\frac{\phi}{3}\right)^{\frac{2\left(1/6\right)^1}{2\cdot 1}}}{\frac{2\cdot 1^1}{2\cdot 1}} = 1.00771588137... \tag{1}$$

where the extra 2 in the numerator of the power is because this is the first dimension of mass, p_m=1, as described above (see Fig. 7). Since all masses operate in three dimensions of space, p_s always equals 3, and $(d_1{}^p/2p_s)$ always equals 1/6. This expression shows the basic form of a hypothetical space-mass function, and is 99.989% accurate to the observed monoisotopic mass of hydrogen. The second mass then has the (1/6) incorporated into the dimension equation for the second dimension, and divided by the equation for the second dimension of mass, as follows:

$$\mathrm{He}_2 = \frac{\left(\frac{\phi}{3}\right)^{\frac{\left(\frac{1}{6}\right)^{p_2}}{2\boldsymbol{p_m}}}}{\left(\frac{\boldsymbol{m}_1{}^{\boldsymbol{p}}}{2\boldsymbol{p_m}}\right)} = \frac{\left(\frac{\phi}{3}\right)^{\frac{\left(\frac{1}{6}\right)^{2}}{2\cdot 2}}}{\left(\frac{1^2}{2\cdot 2}\right)} = \frac{\left(\frac{\phi}{3}\right)^{\frac{\left(\frac{1}{6}\right)^{2}}{4}}}{\frac{1}{4}} = 4.00128124952... \tag{2}$$

where $p_m = 2$ for the second dimension of mass, helium. This model mass is 99.967% of the accurate observed monoisotopic mass of helium, 4.002603254136 [12]. Equation 2 is the DMM form of equation used for all dimensions of mass from 2 through 10 (p=2, 6, 7, 8, 10, 12, 14, 16, 20). Analogously, masses that are deconstructions of dimensions of mass in Fig. 7, up to the 10^{th} dimension of mass, follow the related DMM form of equation that includes $(2p_m+1)$ for deconstructions, instead of simply $2p_m$, (except $Ar_{9\text{-}9}$, which is $2p_m+4$, and only the first isotope of $Cl_{9\text{-}8}$) as follows:

$$\mathrm{Li}_{2\text{-}1} = \frac{\left(\frac{\phi}{3}\right)^{\frac{\left(\frac{1}{6}\right)^{p_3}}{2p_m+1}}}{\left(\frac{m_1^{\,p_3}}{2p_m+1}\right)} = \frac{\left(\frac{\phi}{3}\right)^{\frac{\left(\frac{1}{6}\right)^{3}}{2\cdot 3+1}}}{\left(\frac{1^3}{2\cdot 3+1}\right)} = \frac{\left(\frac{\phi}{3}\right)^{\frac{\left(\frac{1}{6}\right)^{3}}{7}}}{\frac{1}{7}} = 7.00021351065... \qquad (3)$$

where $p_m = 3$ for the third mass. The $2p_m+1$ form of equations applies to $p_m = 3, 4, 5, 9, 11, 13, 15, 17, 19$.

The calculated masses from the 99.989% Solution and observed monoisotopic masses are shown in Table 1, along with the approximation error values. The dimensions of mass give less error than the corresponding deconstructions. Based on this model, the calculated monoisotopic mass for carbon is 12.00000098846…, which is 100.000008% of the observed monoisotopic value of 12.0000000000000, for 0.1 ppm error.

The average error for the first 20 elements, representing the ten dimensions of mass, is 694.0 ppm, with lithium and beryllium showing the greatest deviation.

As mentioned, different possible equations behind the term $(\phi/3)^{1/6}$, as well as the equivalent term $(\pi/3)^{1/6}$, are provided in Electronic Supplementary Material. However, since these cannot be definitively differentiated, and the body of this report is sufficiently challenging, those details are not included here. Nevertheless, the 99.989% Solution presents a new starting point for considering the pattern behind mass and the Periodic Table, using a single form of equations, $2x^p/2p$, $x^p/2p$, and $x^p/(2p+1)$, that are nested into each other to produce approximate monoisotopic masses, in addition to integer masses.

Table 1. Dimension model for mass incorporating Whole PI, using the pattern from dimensions of space.

Element	Number	Dim.-Dec.	Mass	Acc. Mass (A.M.)[a]	Calc. from Model[b]	% A. M.	ppm
H	1	**1**-0	1	1.007825032239	1.0077158813690	99.989	-108.3
He	2	**2**-0	4	4.002603254136	4.0012812495224	99.967	-330.3
Li	3	2-1	7	7.016003436645	7.0002135106505	99.775	-2250.6
Be	4	2-2	9	9.01218306582	9.0000355846361	99.865	-1347.9
B	5	2-3	11	11.0093053645	11.0000059307626	99.916	-844.7
C	6	**3**-3	12	12.000000000000	12.0000009884602	100.000	0.1
N	7	**4**-3	14	14.003074004432	14.0000001647434	99.978	-219.5
O	8	**5**-3	16	15.9949146195717	16.0000000274572	100.032	317.9
F	9	5-4	19	18.9984031627392	19.0000000045762	100.008	84.1
Ne	10	**6**-4	20	19.992440176217	20.0000000007627	100.038	378.1
Na	11	6-5	23	22.989769282019	23.0000000001271	100.045	445.0
Mg	12	**7**-5	24	23.98504169714	24.0000000000212	100.062	623.7
Al	13	7-6	27	26.9815385311	27.0000000000035	100.068	684.2
Si	14	**8**-6	28	27.9769265346544	28.0000000000006	100.082	824.7
P	15	8-7	31	30.973761998427	31.0000000000001	100.085	847.1
S	16	**9**-7	32	31.972071174414	32.0000000000000	100.087	873.5
Cl	17	9-8	35	34.96885268237	35.0000000000000	100.089	890.7
Ar	18	9-9	40	39.962383123724	40.0000000000000	100.094	941.3
K	19	9-10	39	38.963706486449	39.0000000000000	100.093	931.5
Ca	20	**10**-10	40	39.96259086322	40.0000000000000	100.094	936.1

[a]Monoisotopic mass from NIST: http://physics.nist.gov/cgi-bin/Compositions/stand_alone.pl?ele

[b]Calculated from Byrdwell model with (ɸ/3)^(((1/6)^p)/2p) or (ɸ/3)^(((1/6)^p)/(2p+1)), except first dimension of mass, H, which uses (ɸ/3)^((2*(1/6)^p)/2p), and Argon, (ɸ/3)^((2*(1/6)^p)/(2p+4))

3. Discussion

The results above come from a reassessment of the nature of the "unit" that is pi and how it relates to the unit radius that is commonly used in equations for circumference, area, and volume that most people learned as children ($2\pi r$, πr^2, $4/3\pi r^3$), and how those relate to a whole unit circle. Proponents of the use of the symbol τ to represent 2π have long recognized that a single unit of π is only half a circle, and that a whole circle is 2π. In this regard, τ is similar to Whole PI, because both of these represent a whole unit circle. However, the important distinction between τ and ɸ is that τ has a value that is numerically different from π, because it is still based on a unit r. ɸ has the benefit that it is the exact same value as π that has been known for centuries, only the understanding of it is different. The figures herein attempt to make it clear that a single symbol, π, has been associated with two different units, a unit radius and a unit diameter. Because of this, the unit radius or unit diameter had to accompany the symbol for π, as in $2\pi r_1$ or πd_1, otherwise a mathematical paradox could result, such that $2\pi = \pi$. The solution to the problem of having one symbol for two different units is straightforward, though unconventional. By adopting the classic symbol for pi to represent π based on a unit radius, and Whole PI to represent ɸ based on a unit diameter, all ambiguity is eliminated, and the symbol alone clearly identifies the unit on which pi is based.

It is important to differentiate the goals of tauists from the reasons for introducing Whole PI as a hypothesis. Tauists generally seek to simplify equations in physics and other areas by eliminating the "2" that often accompanies π, in the form of 2π. Their goal is to eliminate extraneous 2's that must be followed through complex equations. In contrast, the reason for introducing Whole PI is not to simplify the nomenclature, but to reveal patterns behind the dimensions of space. The purpose is **exposition** rather than abbreviation. Thus, some tauists may take exception to the fact that the first dimension now has two 2's in 2ɸ$d^p/2p$. But examination of the equations above reveals that the 2's come from different places and have different meanings. The two in the denominator reflects the duality of all dimensions, while the two in the numerator identifies the first dimension as unique. If the 2/2 is cancelled out, the indications of duality and the first dimension disappear, and less meaning is conveyed in the simplified equation ɸd, and the common pattern behind all three dimensions is no longer evident.

In looking for a symbol for Whole PI, most Greek letters that contained a circle were already used for other ideas and concepts, such as ϕ, which represents the Golden Ratio [13]. Therefore it was appropriate to develop a new symbol, and this provided the opportunity to reflect the meaning of the symbol with the symbol itself. In fact, the symbol also represents the integer value of ɸ, since the value of ɸ is ~3, and the symbol contains three diameters, |||. Furthermore, the minimum projection of the symbol is one diameter, the defining unit, when viewed from above. The minimum projection of the symbol viewed from the side is two diameters, and when viewed from the front it also exhibits the 2:1 ratio that reflects the fact that each diameter is composed of two radii. The inherent duality of the dimensions is reflected in the equations that use ɸ, since the "2p" in the denominator of every dimension calculates the number of radii present in each dimension. Also, the six radii of three-dimensional space can be seen in the symbol ɸ, because of the way the diameters intersect each other and the circle.

A major hurdle to an entirely new symbol is that it is not included in existing fonts. However, these days producing a new font is straightforward and not very time-consuming, as demonstrated by the Whole PI TrueType font used throughout this report, which took only a couple of hours to produce in a form that is recognized by most programs commonly used. This font is freely available in the electronic supplementary materials and will be updated over time at: http://www.ars.usda.gov/services/software/software.htm?modecode=80-40-05-05.

Another convention has been used here to differentiate π and ɸ. The name for π can appear as pi or Pi, depending on whether it is at the beginning of a sentence or not, or in a title. Therefore, the name for ɸ has been used as all-capital PI, so that it can be distinguished from pi without having to explicitly state Whole PI. Thus, pi and PI have the same spelling and the same numerical value, but a different understanding, in the same way that the symbols have the same value but reflect different definitions of the underlying unit.

Based on the new understanding of PI, new equations were developed that make the pattern behind the equations very clear. In contrast, it is not easy to see that pattern behind the equations that most of us learned as children. $4/3\pi r^3$? Where did the 4/3 come from? Why? If one learned integral calculus, it became possible to see where this came from, but it is not obvious to the vast majority of people who did not learn calculus. On the other hand, the pattern behind the equations using ⦶ is obvious: there are only two forms, one for the first dimension, which is unique, and another for both of the other dimensions, and all forms are identical except for the extra "2" in the equation for the first dimension. Thus, ⦶ not only solves the pi symbol paradox, but also it reveals the pattern behind the dimensions directly.

If the new, straightforward equations for the dimensions of space were the only outcome of the new understanding of Whole PI, that would be sufficient reason for adoption of the new symbol, ⦶. But the fact that equations for masses can be constructed in the same form as the dimensions of space provides additional benefit. In the same way that the equations for dimensions of space contained the dimension number as the power, and the number of radii (the subcomponents of the unit diameter) as the denominator, the equations for mass contain the element number as the power and the number of subcomponents, protons and neutrons, in the denominator. Thus, the new equations reflect the inherent duality in mass. This allows us to think of mass in a new way, as dimensions of mass that follow the same form of equations as dimensions of space.

Many chemists have not considered the fact that the periodic table is based on an empirical definition of the "unit". Originally, the unit was a hydrogen atom set as 1 mass unit. Later, 1/16 of an oxygen atom became the defining unit of the periodic table. And finally, 1/12 of a carbon atom has become the defining unit [14] (https://en.wikipedia.org/wiki/Atomic_mass_unit). The 99.989% Solution based on ⦶ and/or π provided here is not a complete and all-encompassing model for mass to replace the empirical basis, since it does not include factors for isotope distribution, or account for negative mass defects, or other factors mentioned above. It is a converging series that converges with diminishing mass defect. Therefore, it requires additional terms to incorporate the factors discussed above. Nevertheless, it does provide a new foundation for thinking about mass in new ways, and a starting point for developing a non-empirical model for the mass scale

The new model also allows us to consider the anomalies in the periodic table [15], Fig. 8, where an element with a higher atomic number has a lower mass than the preceding element. We can ask "are the masses of elements such as argon anomalously high, or is the mass of the neighboring element anomalously low?". In the case of argon, it is easy to see that it is anomalously high, having a large deconstruction in the equation, $m^{18}/(2p+4)$. In the case of cobalt and nickel, it is less clear. The model reveals that cobalt is $m^{27}/(2p+5)$ in Fig. 8, which is similar to neighboring ^{23}V, ^{25}Mn, and ^{29}Cu, whereas the primary nickel isotope is $m^{28}/(2p+2)$, which is lower than the other transition metals. Thus, the model reveals that the mass of nickel is anomalously low, instead of cobalt being anomalously high. On the other hand, the most abundant isotope of ^{34}Se, mass = 80, arises from $m^{34}/(2p+12)$, which is anomalously high compared to the major isotope of ^{35}Br, mass 79, which is $m^{35}/(2p+9)$ that is similar to surrounding elements. Similarly, ^{52}Te, mass=130, $m^{52}/(2p+26)$, is anomalously high compared to ^{53}I, mass 127, which is $m^{53}/(2p+21)$, similar to neighboring elements.

It is worth mentioning that the dimension model of mass (DMM) and the 99.989% Solution showed three sequential deconstructions of mass that started at a location that was two masses (i.e. at lithium) prior to the observation of the first of the three "p" electronic orbitals that begin with boron. Similarly, the DMM and 99.989% Solution exhibited five alternating unit deconstructions (2p+1), excluding chlorine as discussed above, prior to the beginning of the five "d" orbitals. The last unit deconstruction occurred at element 19, potassium, two elements before the observation of the first "d" orbitals that begin with scandium. Thus, it appears that deconstructions in the DMM occurred prior to observation of patterns in electronic energy levels. It might be anticipated that the higher levels of deconstructions that occur with chlorine and argon (chlorine gave 2p+1 and higher 2p+3 than all prior elements, argon gave 2p+4), may precede and be related to other phenomena that occur later in the table.

4. Conclusions

Presented here is a new hypothesis based on Whole PI to complement, not replace, the classic understanding of pi and the possibly more useful tau. In his “Tau Manifesto” [16], Mike Hartl makes a good argument for the virtue of using τ instead of 2π. It does make sense to refer to a whole circle, instead of half of a circle, as τ does, especially since 2π appears so often in physics and mathematics, and fractions of τ are equal to fractions of a circle. However, since τ is still based on radii, it is also useful to consider the whole circle based on the diameter. While the radius-based circle is commonly used in physics, geometry, trigonometry, and other areas, the equations behind the diameter-based whole circle allow the patterns behind the dimensions to be seen without resorting to integral calculus. Thus, it is appropriate to have three symbols relating to the circle: 1) classic π used in the classic equations for circumference, area, and volume; 2) τ to refer to the whole circle based on radius and eliminate constantly referring to 2π, and 3) ϕ to refer to the whole circle based on diameter, which makes the pattern behind the dimensions more evident at face value.

Since the models for space and mass above both contain the pattern $2x^p/2p$ and $x^p/2p$ for the first and following dimensions, respectively, we can consider the possibility of whether these might apply to other variables. For instance, both $E=mc^2$ and $E=hc/\lambda$ contain the constant “c”, the speed of light in a vacuum. The speed, or velocity, is Δspace/Δtime. Since we already constructed the model for dimensions of space, we could conjecture that if there were such a thing as dimensions of time, they might similarly follow the pattern of $2t^p/2p$ and $t^p/2p$ for the first two dimensions. Finally, every definition, term, equation, and concept discussed in the history of man is the result of thought, so we could also speculate whether there exist dimensions of thought, and construct the appropriate equations using the model presented here.

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Electronic Supplementary Material

The Case for Whole PI and Alternative Equations for Space, Mass, and the Periodic Table

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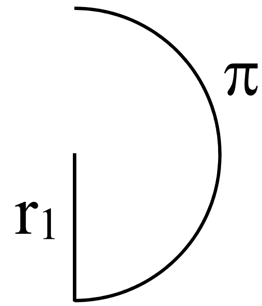
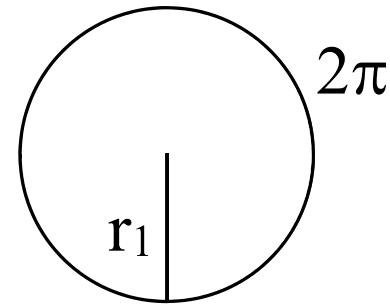
See Supplemental Figure 1 below to demonstrate this analogy.
Imagine a wife calls her husband at work as says: "Honey, would you please pick up a jug of milk on the way home?"
He responds, "Sure, no problem." He stops by the store on the way home and picks up a half-gallon jug of milk.
When he gets home, she says "No, No, I needed a GALLON of milk!"

The next day, the wife calls and says: "The kids already went through that jug of milk you bought yesterday, would you buy a GALLON of milk this time?"
He responds, "Sure, no problem." He stops by the store on the way home and picks up TWO half-gallon jugs of milk.
When he gets home, she says "Yes, you brought a gallon of milk home, but I needed it in one jug instead of two jugs, because we wanted to use the empty jug to make a terrarium later."
He responds: "Honey, you told me to bring home a GALLON of milk, so I brought home a gallon of milk. You didn't tell me what units to bring the milk home in. A gallon is a gallon, whether it is made up of two half-gallons or one whole gallon. They are entirely equivalent, and two half-gallons make one gallon every day of the week."

There are times when we want gallons of milk in half-gallon jugs, and there are times when we want gallons of milk in whole gallon jugs. The term jug needs to be specified based on the unit on which it is based (half-gallon jug or gallon jug). Seeing the word "jug" alone is not sufficient to tell which jug is being referred to. Similarly, a gallon can be obtained from two jugs or one jug, depending on the unit definition of the jug. To simply say that "a gallon is a gallon, regardless of the units it comes in, and they are entirely equivalent" belies the fact that there are times when one unit is more useful than the other, and although we can easily convert from one unit to the other (2 x "half-gallon jug" = "gallon jug"), they are not completely functionally equivalent (it is easier to make a one gallon terrarium in a gallon jug than two half-gallon jugs).

2 x  = 

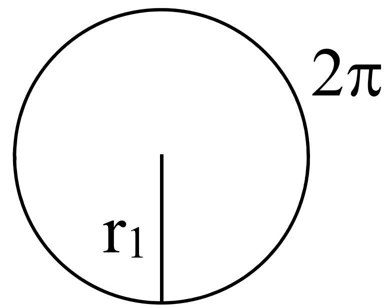
2 x 

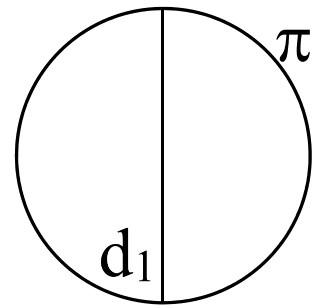
= 

2 x “1 jug” = 1 gallon

Unit Circle

 = 



= 

2 x “1 jug” = “1 jug” = 1 gallon

Unit Circle

$C = 2\pi \cdot r_1$ $C = \pi \cdot d_1$

2 x “1 jug_{HG}” = “1 jug_{G}”

$2r_1 = d_1$

Supplement to section 2.3.

It may be instructive to briefly examine the form of $(\phi/3)^{1/6}$, where ϕ is divided by three dimensions of space, $3d_1$, raised to the power of $d_1^p/2p_s$. to give

$$\left(\frac{\phi}{3}\right)^{\frac{1}{6}} = \left(\frac{\phi}{3d_1}\right)^{\left(\frac{d_1^{p}}{2p_s}\right)} \quad \text{(Eq. ESM 1)}$$

where again $p_s = 3$, for the three dimensions of space in which mass operates. In this case, ϕ can be used instead of π, because ϕ is being divided by three diameters, which are proportional to the three dimensions of space. This starting point only incorporates space, but when the power (1/6) is nested into the dimensions of mass equations and divided by the dimension+deconstruction forms of equations for mass, it becomes a space-mass function.

An alternative way to model the unit mass in the 99.989% Solution is to use $(\pi/3)^{1/6}$. Although it was at first empirically determined, the constant $(\pi/3)^{1/6}$ can be modeled using the principles presented in the body of this report. As a first approximation, $(\pi/3)^{1/6}$ is mathematically equal to the first dimension of mass, $2m^p/2p_m$, times the third dimension of space, $\phi d^p/2p_s$, times the factor $2\pi/\phi$, all raised to the power of $2m^p/2p_m$ times $d^p/2p_s$ as follows:

$$\left(\frac{\boldsymbol{\pi}}{3}\right)^{\frac{1}{6}} = \left(\frac{2\boldsymbol{\pi}}{\phi}\cdot\frac{2\boldsymbol{m}_1^{\boldsymbol{p}_1}}{2\boldsymbol{p_m}}\cdot\frac{\phi\boldsymbol{d}_1^{\boldsymbol{p}_3}}{2\boldsymbol{p_s}}\right)^{\left(\frac{2\boldsymbol{m}_1^{\boldsymbol{p}_1}}{2\boldsymbol{p_m}}\cdot\frac{\boldsymbol{d}_1^{\boldsymbol{p}_3}}{2\boldsymbol{p_s}}\right)} \quad \text{(Eq. ESM 2)}$$

where the factor $2\pi/\phi$ is incorporated because, for the defining unit, we are interested in a single unit mass, proportional to 'p_m', instead of the proton + neutron pairs represented by '$2p_m$'. We went to great lengths to differentiate π based on sub-components (radii) and ϕ based on duality and the sum of sub-components (diameters) in the text. Thus, since π is directly proportional to radii, and not diameters, as seen in Fig. 2a, it appears that π may be appropriate to be proportional to one sub-component (one proton) that constitutes the unit mass, rather than ϕ, which might be thought of as proportional to the sum of sub-components, a proton + neutron. Furthermore, this form of model for the unit mass has the advantage that it incorporates both the first dimension of mass ($2m^p/2p$) and three dimensions of space ($\phi d^p/2p$), such that it is a "1mass3space" function. There are other conceivable ways to model the 99.989% solution, but they all lead to the same value for the unit "1mass3space" of 1.00771588137…